(1.4) Function or Mapping

* **Def^n(1):** If S and T are two non empty sets, then a function from S to T is a rule, denoted as $f: S \to T$, which associates to each element $x \in S$, a unique element $y \in T$; the element y, written as $y = f(x)$ is called the image of x and x is called a pre-image of y. The set S is called the domain and T the co-domain of the function $f: S \to T$.
The set $f(S) = \{f(x) : x \in S\}$ is called the range of the function $f: S \to T$.

Illustrations:

1. $f: N \to N$ defined by $f(n) = n^2$ is a function whose range is $\{1, 4, 9, 16, 25, \cdots\}$

2. $f: N \to I$ defined by $f(n) = (-1)^n$ is a function whose range is $\{-1, 1\}$.

(3) $f(x) = \sqrt{a^2 - x^2}$; $-a \le x \le a$, is a function whose domain $\{x : -a \le x \le a\}$ and range is the set of positive real numbers.

* **Def^n(2):** A mapping $f: S \to T$ is called one-to-one, if

$$x \neq y \Rightarrow f(x) \neq f(y) \quad ; x, y \in S$$

OR

$$f(x) = f(y) \Rightarrow x = y \quad ; x, y \in S.$$